Strength of Materials

Through Questions & Answers

For **ESE, GATE, PSUs**

& Other Competitive Examinations

by

Dr. U.C. Jindal

M.Tech, Ph.D.

Former Professor & Head of the Department

Department of Mechanical Engineering

Delhi College of Engineering, Delhi

MADE EASY PUBLICATIONS

www.madeeasypublications.org

MADE EASY Publications Pvt. Ltd.

Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016
Contact: 9021300500
E-mail: infomep@madeeasy.in
Visit us at: www.madeeasypublications.org

Strength of Materials through Questions & Answers : Dr. U.C. Jindal

First Edition: 2014
Reprint : 2015
Reprint : 2016
Second Edition (Revised): 2018
Reprint : 2022
Reprint : 2023

Reprint : 2024

PREFACE

I must thank CMD of MADE EASY Group, **Mr. B. Singh** for providing me an opportunity to reach out to the Student Community at large through my present book **"Strength of Materials through Questions & Answers".** Students may be benefitted from my 50 years of teaching /research experience through this book.

Questions in the book are designed on the pattern of questions that are being asked in university examinations and competitive examinations of UPSC/GATE/PSUs.

The book has been thoroughly and questions from competitive examinations for the last 2 years have been added, in this revised and enlarged edition.

Further improvements in the text book will be made after getting the response from the students.

Any error in printing or calculations pointed out by the reader will be acknowledged with thanks by the author.

Dr. U. C. Jindal
Author

CONTENTS

Simple Stresses and Strains

CHAPTER 1

In this chapter we will discuss various types of external loads that can be applied on a body and the deformation produced in body. Due to the applied load, stresses and strains are developed in a body. Relationship between different types of stresses and different types of strains will be developed.

Positive and negative normal stresses and positive and negative shear stresses will also be described.

Question 1.1 What do you understand by an axial load on a body and axial stress developed in a body.

Solution: Consider a circular bar *AB*, of diameter d and axial length *L* as shown in figure 1.1. End *A* of the bar is fixed and a load *P* is applied along 00 axis of the bar. **Axis 00 passes through the centroids of all the sections of the bar, as shown.** Load *P* is perpendicular to all sections of the bar. The load applied can be a point load along the axis 00 or a uniformly distributed load over the section of the bar. To maintain equilibrium, a reaction *R* = *P* is developed at the fixed end.

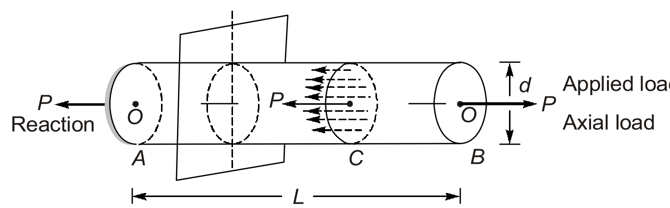


Fig. 1.1

If you consider a portion BC of the bar *AB*, then a force *P* at section *B* is resisted by equal and opposite force at section *C*, this equal and opposite force *on internal section* is known as internal resistance. **This internal resistance *P* per unit area is defined as stress.**

or stress, $$\sigma = \frac{P}{\text{area of section}} = \frac{4P}{\pi d^2}$$

Note that force *P* is perpendicular to section.

Therefore stress σ is normal to the section.

Stress σ is perpendicular to the section of the bar, and is defined as normal stress. Direct stress is along the axis of bar. If axial load *P* is expressed in N (newton) and area of cross-section in mm^2, then units of stress are N/mm^2 (newton per mm^2). Stress is a second order vector, it has both magnitude and direction.

Moreover load is normal to the section and its direction is away from the plane. Note that load *P* is normal to plane *and is pointing away from the plane*.

Similarly internal resistance *P* is normal to plate at *C* and is pointing away from the plane. **This type of load is known as tensile load and stress produced by tensile load is tensile stress (or positive normal stress).**

Segments 1 and 2 have cross-sectional area of 100 mm^2 and 60 mm^2. Young's modulus of 2×10^5 MPa and 3×10^5 MPa, and length of 400 mm respectively. The strain energy (in N-mm up to one decimal place) in the bar due to the axial load is ________.

[CE, GATE : 2017 (set-1)]

Solution:

$$\sigma_1 = \frac{1000}{100} = 10 \text{ MPa}$$

$$\sigma_2 = \frac{1000}{60} = 16.667 \text{ MPa}$$

$$V_1 = 100 \times 400 = 4 \times 10^4 \text{ mm}^3$$

$$V_2 = 60 \times 900 = 5.4 \times 10^4 \text{ mm}^3$$

$$\text{Strain energy} = \frac{10^2}{2 \times 2 \times 10^5} \times 4 \times 10^4 + \frac{(16.667)^2}{2 \times 3 \times 10^5} \times 5.4 \times 10^4$$

$$= 10 + 25 = 35 \text{ Nmm}$$

Practice Q.1.42 A wrought iron bar of circular section, diameter 12.5 mm gauge length of 100 mm is tested in tension and following observations are made:

(a) yield load 29.6 kN
(b) maximum load 44.8 kN
(c) load at fracture 37 kN
(d) diameter at neck 9.1 m
(e) total extension in sample 27.6 mm

Determine

(i) yield strength
(ii) ultimate tensile strength
(iii) actual breaking strength
(iv) percentage elongation
(v) percentage reduction in area

Ans. [241.2 N/mm^2, 365.06 N/mm^2, 568.9 N/mm^2, 27.6%, 47%]

Objective Type Questions

Q.1 A circular tapered bar A tapers from diameter $2d$ to d over an axial length L. It is subjected to axial load P. Another bar B of same length, same material but of uniform diameter 1.5 d is also subjected to same load. What is the ratio of strain energy absorbed by A over strain energy absorbed by B, i.e, U_A/U_B

(a) 1.333
(b) 1.125
(c) 0.888
(d) 0.750

Q.2 A steel wire of area of cross-section 3 mm^2, length 3 m, is supported through a pulley. The ends of the wire are fixed in ceiling. A load of 600 N is hung at the centre of the pulley. If E for steel is 200 GPa, by how much distance, centre of the pulley comes down

(a) 3 mm
(b) 1.5 mm
(c) 0.75 mm
(d) 0.3 mm

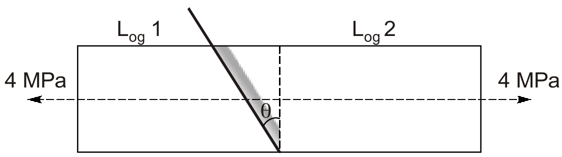
Q.3 A bar is subjected to uniaxial tension. Axial strain in bar is ε. While volumetric strain in bar is 0.4 ε. What is Poisson's ratio of the material

(a) 0.4
(b) 0.33
(c) 0.30
(d) 0.25

Q.4 Total extension in steel bar shown in figure is equal to, if A = area of cross-section, E = 200 kN/mm^2

The tension in the string are
(a) $T_1 = 100$ N, $T_2 = 0$ N
(b) $T_1 = 0$, $T_2 = 100$ N
(c) $T_1 = 75$ N, $T_2 = 25$ N
(d) $T_1 = 25$ N, $T_2 = 75$ N

Q.38 A carpenter glues a pair of cylindrical wooden logs by bending the end faces at an angle of $\theta = 30°$ as shown in the figure.



The glue used at the interface fails if
Criterion 1: The maximum normal stress exceeds 2.5 MPa
Criterion 2: The maximum shear stress exceeds 1.5 MPa

Assume at the interface fails before the logs fail, when a uniform tensile stress of 4 MPa is applied the interface
(a) Fails only because of criterion 1
(b) Fails only because of criterion 2
(c) Fails because of both criterion 1 and 2
(d) does not fail

Answers

1. (b)	2. (c)	3. (c)	4. (c)	5. (a)
6. (b)	7. (d)	8. (c)	9. (c)	10. (d)
11. (c)	12. (a)	13. (b)	14. (a)	15. (b)
16. (d)	17. (b)	18. (b)	19. (a)	20. (b)
21. (b)	22. (d)	23. (a)	24. (c)	25. (b)
26. (a)	27. (c)	28. (a)	29. (b)	30. 0.5
31. (d)	32. (a)	33. (a)	34. (c)	35. (d)
36. (b)	37. (b)	38. (c)		

Explanations

1. (b)

Tapered bars, $dL_A = \dfrac{4PL}{\pi E2d \times d} = \dfrac{4PL}{\pi E2d^2}$

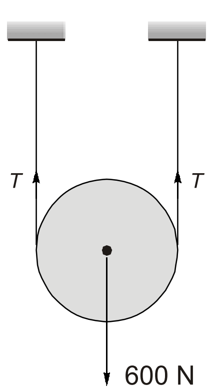
Uniform bar, $dL_B = \dfrac{4PL}{\pi E2.25d^2}$

$$U_A = \frac{1}{2}PdL_A$$

$$U_B = \frac{1}{2}PdL_B$$

$$\frac{U_A}{U_B} = \frac{2.25}{2} = 1.125$$

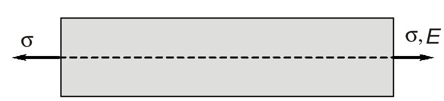
2. (c)



$$T = 300 \text{ N}$$
$$A = 3 \text{ mm}^2$$
$$\sigma = \frac{300}{3} = 100 \text{ N/mm}^2$$
$$\epsilon = \frac{100}{200000} = \frac{1}{2000}$$
$$\delta l = \epsilon L = \frac{3000}{2000} = 1.5 \text{ mm}$$

Vertical displacement of pulley $= \dfrac{dl}{2} = 0.75$ mm

3. (c)



$$\text{Axial strain} = \varepsilon$$
$$\text{Poisson's ratio} = \nu$$
$$\text{Lateral strain} = -\nu\varepsilon$$
$$\text{Total volumetric strain} = \varepsilon - \nu\varepsilon - \nu\varepsilon$$
$$= (1 - 2\nu)\varepsilon$$
$$(1 - 2\nu)\varepsilon = 0.4\varepsilon$$

or Poisson's ratio, $\nu = 0.3$

Principal Stresses and Strains

CHAPTER 3

Determination of principal stresses at the critical section of any structure or any engineering component is the most important step in experimental stress analysis of that structure or engineering component. In this chapter we will learn about (i) analytical determination of principal stresses (ii) graphical method of determining principal stresses (iii) determination of principal strains.

Question 3.1 What are principal stresses? What is the importance of principal stresses?

Solution: At any point in a stressed body, there exists a set of 3 planes perpendicular to each other, such that on these 3 planes, there are only normal stresses say p_1, p_2, p_3 and no shear stress exists on these planes. These three normal stresses are called principal stresses and three planes on which principal stresses act are called principal planes. One of these principal stresses, is the maximum most normal stress at the point. Determination of this maximum principal stress is important for designing any structure or any engineering component.

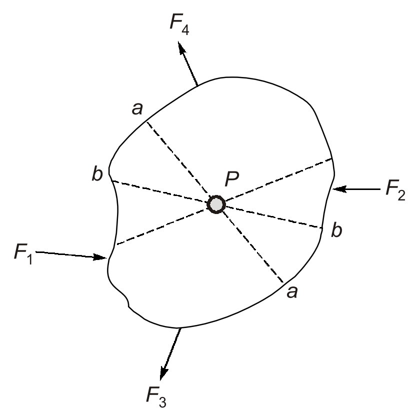
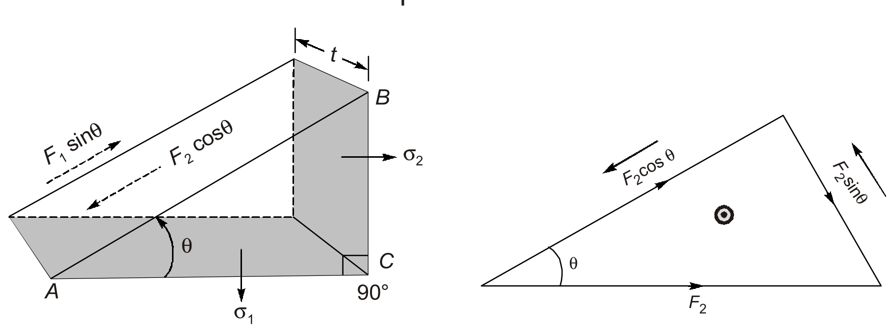


Fig. 3.1

Fig. 3.1 shows a body subjected to external forces, F_1, F_2, F_3, F_4 etc. Say P is the critical point, through this point innumerable number of planes such as *aa*, *bb* can pass, but there is a set of 3 orthogonal planes on which only the normal stresses act and shear stress is absent on these planes, then these planes are called principal planes and normal stresses on these planes are called principal stresses.

Question 3.2 Figure 3.2 shows a triangular solid block ABC, with thickness t. Planes AC and BC are perpendicular to each other. On plane AC, normal stress σ_1 acts and on plane BC, normal stress σ_2 acts. Determine normal and shear stresses on inclined plane AB, which is inclined with plane AC at an angle θ. Plane AC can be taken as a reference plane



(a) $\tau_{xy} = \dfrac{\sigma_x + \sigma_y}{2}$ (b) $\tau_{xy} = \dfrac{\sigma_x - \sigma_y}{2}$

(c) $\tau_{xy} = \sqrt{\sigma_x \times \sigma_y}$ (d) $\tau_{xy} = \sqrt{\sigma_x^2 + \sigma_y^2}$

Q.5 In a strained material, on two mutually perpendicular planes only the normal stress $+\sigma$ and $-\sigma$ act. What is the normal stress on a plane inclined at 45° to the plane of $+\sigma$ and $-\sigma$

(a) 2σ (b) σ

(c) $\dfrac{\sigma}{2}$ (d) zero

Q.6 At a point, if p_1 = maximum principal stress, p_2 = minimum principal stress and τ_{max} = maximum shear stress and ratio p_1/p_2 = 2.5. Then what is the ratio of p_1/τ_{max}

(a) 2.0 (b) $\dfrac{7}{3}$

(c) $\dfrac{10}{3}$ (d) $\dfrac{11}{3}$

Q.7 Principal stresses at a point are +160 MPa, 120 MPa. If Poisson's ratio of material is 0.35. What is the ratio of principal strains $\varepsilon_1/\varepsilon_2$

(a) 1.148 (b) 1.333

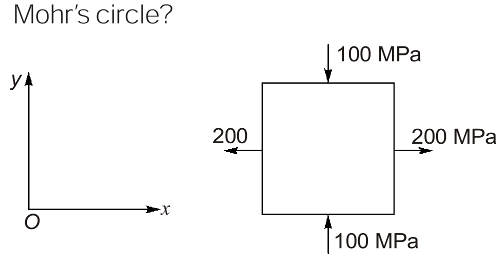
(c) 1.84 (d) 1.96

Q.8 At a point in a two dimensional state of strain there is pure shearing strain of magnitude γ_{xy} radians. Which one of the following is the maximum principal strain

(a) γ_{xy} (b) $\dfrac{\gamma_{xy}}{\sqrt{2}}$

(c) $\dfrac{\gamma_{xy}}{2}$ (d) $2\gamma_{xy}$

Q.9 Consider a two dimensional state of stress given for an element as shown in the diagram given below. What are the coordinates of the centre of Mohr's circle?



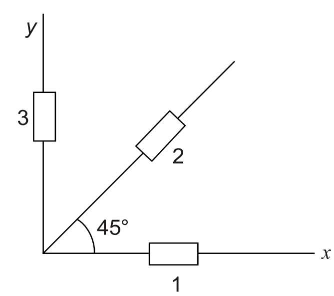
(a) (0.0) (b) (100, 200)

(c) (200, 100) (d) (50, 0)

[IES 2004]

Q.10 A rectangular strain gage rosette, shown in figure, gives following readings in a strain measurement, which are

$\varepsilon_1 = 1000 \times 10^{-6}$, $\varepsilon_2 = 800 \times 10^{-6}$ and $\varepsilon_3 = 600 \times 10^{-6}$



The direction of principal strain with respect to gauge 1 is

(a) 0° (b) 15°

(c) 35° (d) 45°

[IES 20111]

Q.11 If the principal stresses and maximum shearing stresses are of equal numerical value at a point in a stress body. The state of stress can be termed as

(a) isotropic

(b) uniaxial

(c) pure shear

(d) generalised plane stress

[IES 2010]

Q.12 A shaft with a circular cross-section is subjected to pure twisting moment. The ratio of the maximum shear stress to the largest principal stress is

(a) 2.0 (b) 1.0

(c) 0.5 (d) 0

Q.13 The state of stress on an element is as shown in the figure. If $E = 2 \times 10^5$ N/mm^2 and Poisson's ratio = 0.3, the magnitude of the stress σ for no strain in BC is

The Young's modulus of the material is 2×10^{11} N/m^2, and Poisson's ratio 0.3. If σ_{zz} is negligible and assumed to be zero, what is strain ε_{zZ}

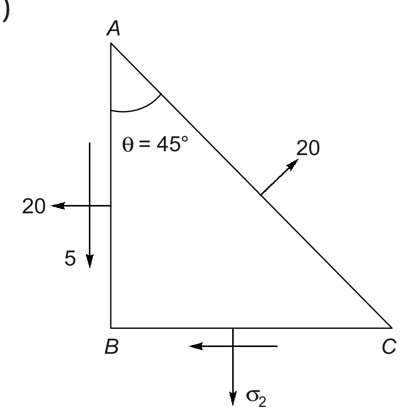
(a) -120×10^{-6} (b) -60×10^{-6}
(c) 0.0 (d) $+120 \times 10^{-6}$

Answers

1. (c)	2. (d)	3. (b)	4. (c)	5. (d)
6. (c)	7. (c)	8. (c)	9. (d)	10. (a)
11. (c)	12. (b)	13. (d)	14. (a)	15. (c)
16. (d)	17. (c)	18. (1.0476×10^{-3})	19. (d)	
20. (c)	21. (a)	22. (b)	23. (b)	24. (c)
25. (d)	26. (a)	27. (a)	28. (d)	29. (d)
30. (c)	31. (b)	32. (b)	33. (d)	34. (c)
35. (c)	36. (a)			

Explanations

1. (c)



Say σ_2 is normal stress as plane *BC*, then

$$20 = \frac{20+\sigma_2}{2} + \frac{20-\sigma_2}{2}\cos 90° + 5\sin 90° = 10 + \frac{\sigma_2}{2} + 5$$

$$\sigma_2 = 10 \text{ N/mm}^2$$

$$\tau_\theta = \frac{\sigma_1-\sigma_2}{2}\sin 2\theta - \tau\cos 2\theta = \frac{20-10}{2}\sin 90° - 5\cos 90°$$
$$= +5 \text{ N/mm}^2$$

$$p_{max} = \frac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$$
$$= 15 + \sqrt{5^2+5^2}$$
$$= 22.07 \text{ N/mm}^2$$

2. (d)

$$p_{max} = \frac{\sigma_1+\sigma_2}{2} + \tau_{max}$$

$$p_{min} = \frac{\sigma_1+\sigma_2}{2} - \tau_{max}$$

$$p_{max} - p_{min} = 2\,\tau_{max} \quad \text{but } p_{max} = 2p_{min}$$

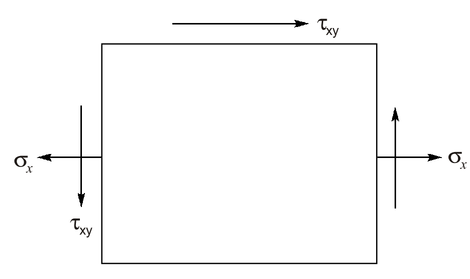
$$p_{min} = \frac{1}{2}p_{max}$$

So $\frac{1}{2}p_{max} = 2\tau_{max}$

$$p_{max} = 4\,\tau_{max}$$
$$= 4 \times 50 = 200 \text{ N/mm}^2$$

3. (b)

$\tau_{xy} \neq 0,\ \sigma_x \neq 0,\ \sigma_y =$



4. (c)

$$p_{min} = 0 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

or $$\frac{\sigma_x+\sigma_y}{2} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

or $$\left(\frac{\sigma_x+\sigma_y}{2}\right)^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + +\tau_{xy}^2$$

or $$\sigma_x\,\sigma_y = \tau_{xy}^2$$

or $$\tau_{xy} = \sqrt{\sigma_x \times \sigma_y}$$

5. (d)

Thin Cylindrical and Spherical Shells

CHAPTER 5

Any shell cylindrical or spherical when subjected to external or internal fluid pressure, stresses are developed in the wall of the shell. If the variation in radial and hoop stresses along the radial thickness of the shell is negligible, then it is termed as a thin shell. **Generally if the ratio of D/t i.e., diameter/thickness is greater than 20, then shell is classified as a thin shell.** In such a shell, radial stress is much less than the hoop stress. Therefore effect of radial stress in the calculation of strains is generally neglected. In a cylindrical shell, if ratio of D/t is equal to 20, then hoop stress is 10 times the radial stress, i.e., pressure p.

Question 5.1 **Consider a thin cylindrical shell, of diameter, D thickness, t subjected to internal pressure p and derive expression for axial and circumferential stresses developed in shell.**

Solution:

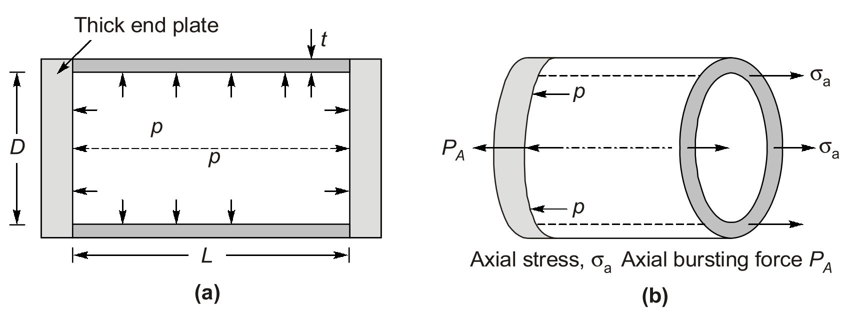


Fig. 5.1

Fig. 5.1 (a) shows a thin cylindrical shell of internal diameter D, wall thickness t, length L, with thick end plates. Cylindrical shell is subjected to internal pressure p. The internal pressure may be developed in cylinder by pumping extra volume δV of liquid in addition to original volume $V\left(=\frac{\pi}{4}D^2L\right)$. Due to internal fluid pressure:

(a) Length L, of the cylinder will increase, developing axial tensile stress in cylinder

(b) Diameter D, of the cylinder will increase or consequently the circumference of the cylinder will increase, developing hoop tensile stress.

Axial Stress: Due to internal pressure, axial bursting force P_A developed in cylinder is

Axial bursting force, $$P_A = p\times\frac{\pi}{4}D^2 \qquad \text{...(i)}$$

imposed along the axial direction as shown in the figure. Assume that the state of stress in the wall is uniform along its length. If the magnitude of axial and circumferential components of stress in the can are equal, the pressure (in MPa) inside the can is ________ (correct to two decimal place).

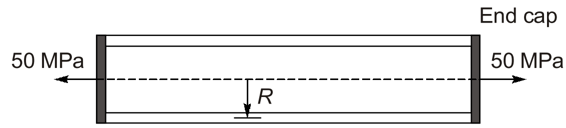


Fig. 5.13

Solution:

$$\sigma_a + 50 = \sigma_c$$
$$\sigma_c = 2\sigma_a$$
$$\sigma_a + 50 = 2\,\sigma_a$$
$$\sigma_a = 50 \text{ MPa} = \frac{pR}{2t}$$
$$p = \frac{50 \times 2 \times 5}{100} = 5 \text{ MPa}$$

Objective Type Questions

Q.1 Hoop stress and longitudinal stress in a boiler shell under internal pressure are 100 MN/m^2 and 50 MN/m^2 respectively. Young's modulus of elasticity and Poisson's ratio of the shell material are 200 GN/m^2 and 0.3 respectively. The hoop strain in boiler shell is

(a) 0.425×10^{-3} (b) 0.5×10^{-3}
(c) 0.585×10^{-3} (d) 0.75×10^{-3}

[IES 1996]

Q.2 From the design point of view, spherical pressure vessels are preferred over cylindrical pressure vessels, because they

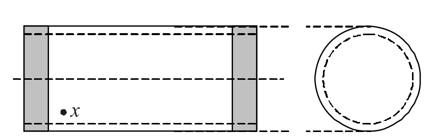
(a) are cost effective in fabrication
(b) have uniform higher circumferential stress
(c) uniform lower circumferential stress
(d) have a larger volume for the same quantity of material used

Q.3 When a thin cylinder of diameter d and thickness t is pressurized with an internal pressure of p (1/m is the Poisson's ratio) and E is the modulus of elasticity, then out of the following, which statement is correct

(a) circumferential strain is equal to $\frac{pd}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)$
(b) longitudinal strain will be equal to $\frac{pd}{2tE}\left(1-\frac{1}{2m}\right)$
(c) the longitudinal stress is equal to $\frac{pd}{2t}$
(d) ratio of longitudinal strain to circumferential strains is equal to $\frac{m-2}{2m-1}$

[IES 1998]

Q.4 A thin cylinder with closed ends is subjected to internal pressure and supported at ends as shown in figure. What is the state of stress at point x?



Q.15 A thin cylindrical pressure vessel with closed ends is subjected to internal pressure. The ratio of circumferential (hoop) stress to the longitudinal stress is

(a) 0.25 (b) 0.50

(c) 1.0 (d) 2.0

[GATE 2015]

Answers

1. (a)	2. (c)	3. (d)	4. (a)	5. (c)
6. (c)	7. (c)	8. (d)		
9. (109.80, 40.2 N/mm²)		10. (d)	11. (b)	
12. (c)	13. (c)	14. (b)	15. (d)	

Explanations

1. (a)

$$\varepsilon_c = \frac{\sigma_c}{E} - \frac{\nu\sigma_a}{E} = \frac{100 - 0.3\times 50}{200\times 1000}$$

$$= \frac{85}{200} = 0.425\times 10^{-3}$$

2. (c)

Have uniform lower circumferential stress

$$(\sigma_c)_{spherical} = \frac{1}{2}\sigma_{c(cylindrical)}$$

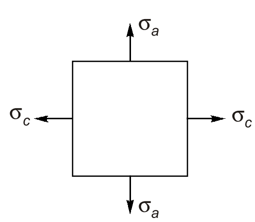
Spherical vessels are costly to fabricate.

3. (d)

$$\varepsilon_a = \frac{pD}{4tE}\left(1 - \frac{2}{m}\right), \quad \varepsilon_c = \frac{pD}{4tE}\left(2 - \frac{1}{m}\right)$$

$$\frac{\varepsilon_a}{\varepsilon_c} = \frac{m-2}{2m-1}$$

4. (a)



No. shear stress, τ

Hoop and axial stresses act, both are tensile.

5. (c)

At the surface

$$\sigma_a = \sigma_0$$
$$\sigma_c = 2\sigma_0$$
$$p = 0 \text{ (radial stress)}$$

$$\tau_{max} = \frac{2\sigma_0 - 0}{2} = \sigma_0$$

6. (c)

$$\sigma_c = 9000 = \frac{500\times 600}{2\times t}$$
$$t = 16.66 \simeq 17 \text{ mm}$$

7. (c)

$$\varepsilon_c = \frac{2\pi(r+u) - 2\pi r}{2\pi r} = \frac{u}{r}$$

8. (d)

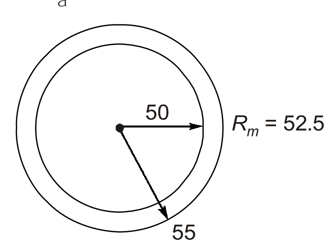
$$\sigma_{all} = \frac{pD}{4t}$$

$$p = \frac{4t\sigma_{att}}{D} = \frac{4\times 1.5\times 45}{1500} = 1.8 \text{ MPa}$$

9. (109.80, +40.2 N/mm²)

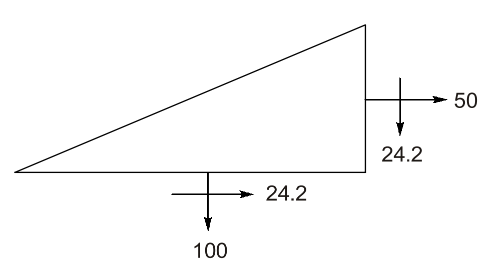
$$\sigma_c = \frac{10\times 100}{2\times 5} = 100 \text{ MPa} \quad \text{...(i)}$$

$$\sigma_a = 50 \text{ MPa} \quad \text{...(ii)}$$



$$J = 2\pi \times 5 \times 52.5^3 = 4.546 \times 10^6 \text{ mm}^4$$

$$\tau_{max} = \frac{2\times 10^6}{4.546\times 10^6}\times 55 = 24.2 \text{ N/mm}^2$$



Thick Shells

CHAPTER 6

In thick shells, the wall thickness is considerable in comparison to inner radius. There is large variation of radial stress and hoop stress along the thickness, in such shells. In thick cylindrical shells, axial stress is independent of radius and remains constant from inner radius to outer radius of the shell.

Thick tubes are used for the transmission of very high pressure as high as 1000 atmospheres as in this case of pressure gauges for the measurement of pressures in transmission lines which are under high pressure liquids.

Question 6.1 Consider a thick cylindrical shell with inner radius R_1, outer radius R_2, subjected to internal pressure p. Derive expressions for radial stress σ_r and hoop stress σ_c starting from basic principles. Clearly state the assumptions used.

Solution:

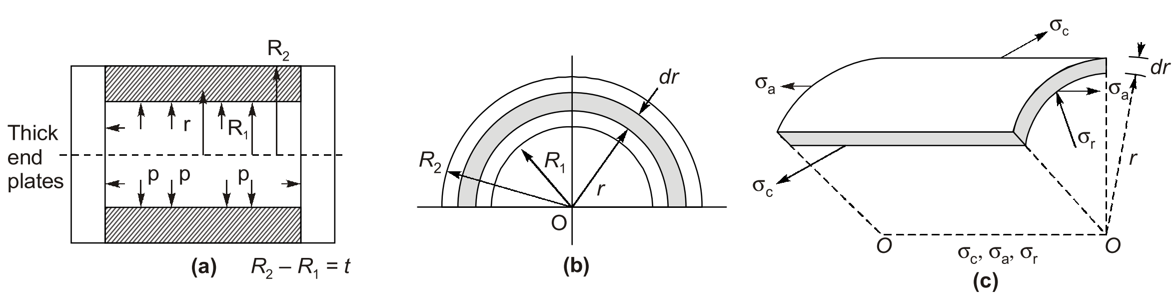


Fig. 6.1

A thick cylindrical shell, with inner radius R_1, outer radius R_2 subjected to internal pressure p is shown in figure 6.1 (a).

There are thick plates at the ends. Under pressure p, cylinder tends to expand introducing σ_a, tensile axial stress and σ_c tensile circumferential stress in the shell. To develop expressions for σ_r and σ_c, following assumptions are taken

(i) Axial strain ε_a, remains constant, along the length of the shell

(ii) No distortion of end plates

Say at a particular radius r, the stresses are σ_c, σ_r, σ_a

These are three principal stresses, perpendicular to each other. If E is Young's modulus and ν is Poisson's ratio, then axial strain,

$$\varepsilon_a = \frac{\sigma_a}{E} + \frac{\nu\sigma_r}{E} - \frac{\nu\sigma_c}{E}$$, Note σ_r is compressive

$$= \frac{\sigma_a}{E} + \frac{\nu}{E}(\sigma_r - \sigma_c) \quad \text{...(i)}$$

As per assumptions, ε_a is constant, E and ν are elastic constants.

Moreover σ_a, axial stress $= \dfrac{p(\pi R_1^2)}{\pi(R_2^2 - R_1^2)} = \dfrac{pR_1^2}{R_2^2 - R_1^2}$, is also constant.

From eq. (i) it is obvious that

$\sigma_c - \sigma_r$ = difference of radial and hoop stresses is constant at any radius

$\sigma_c - \sigma_r = 2A$ (say)

Consider unit length of cylinder say at a particular radius r radial stress is σ_r and at radius is $r + dr$, radial stress is $\sigma_r + d\sigma_r$. Consider equilibrium of elementary ring as shown in fig. 6.2

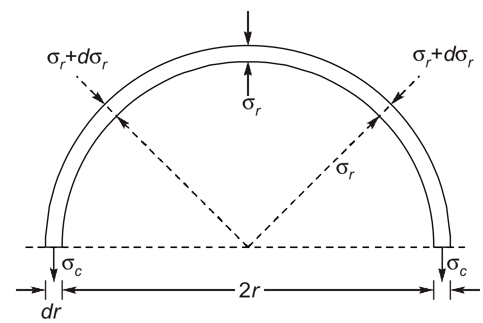


Fig. 6.2

$$2(\sigma_r + d\sigma_r)(r + dr) + 2\sigma_c\, dr = 2\sigma_r r$$

$$\sigma_r r + r d\sigma_r + \sigma_r dr + d\sigma_r dr + \sigma_c\, dr = \sigma_r r$$

Neglecting very small quantities $d\sigma_r\, dr$

We get $\quad r d\sigma_r + \sigma_r dr + \sigma_c dr = 0$

or $\quad r d\sigma_r + (\sigma_c + \sigma_r) dr = 0$

or
$$\sigma_c + \sigma_r = -r\frac{d\sigma_r}{dr} \quad \text{...(ii)}$$

From eq. (i) and (ii)

$$2\sigma_r = -r\frac{d\sigma_r}{dr} - 2A$$

$$2(A + \sigma_r) = -r\frac{d\sigma_r}{dr}$$

or
$$\frac{d\sigma_r}{A + \sigma_r} = -2\frac{dr}{r} \quad \text{...(iii)}$$

Integrating eq. (iii)

$$\ln(A + \sigma_r) = -2 \ln r + \ln B$$

Where B is another constant

or
$$A + \sigma_r = \frac{B}{r^2}$$

Radial stress,
$$\sigma_r = \frac{B}{r^2} - A \quad \text{...(iv)}$$

Hoop stress,
$$\sigma_c = \sigma_r + 2A = \frac{B}{r^2} + A$$

These two equations are known as Lame's equations and constants A and B are Lame's constants.

At any radius $\quad \sigma_c + \sigma_r = 2A$

Both σ_c and σ_r depend on the radius r. Using boundary conditions, constants A and B are determined. For both σ_r and σ_c, hyperbolic curves give the stress distribution along radial thickness of shell.

Theories of Failure

CHAPTER 15

For the prediction of failure of a structural member or a machine component, there are various theories of failure developed by various Professors and Scientists. However these theories are based on principal stresses (their relative magnitudes) at the point of interest of the body. Moreover the use of these theories depends on the mechanical properties of the material of the structural member or machine component.

These mechanical properties are brittleness or ductility; yield strength, ultimate strength, behaviour under tension loading and or compression loading etc.

Question 15.1 What are ductile and brittle materials? Explain with 5 examples of each material. What are the applications of these materials?

Solution:

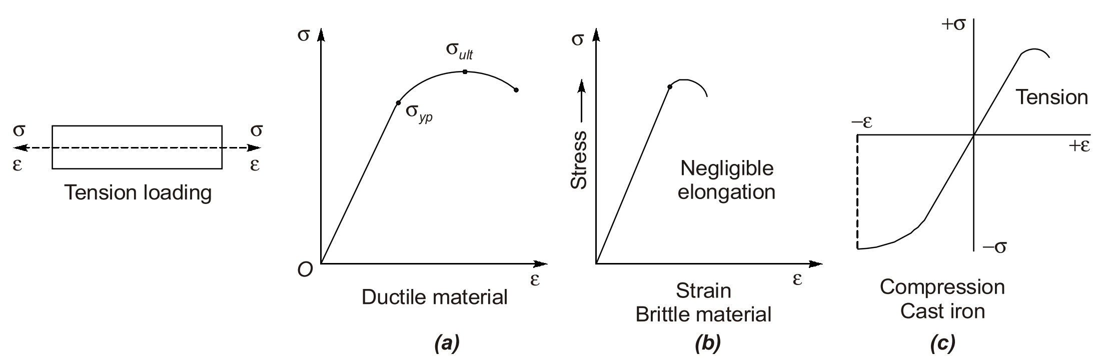


Fig. 15.1

When a material is tested in tension and a graph between load-extension or tensile stress-tensile strain is drawn then a ductile material exhibits sufficient elongation as shown in figure 15.1(a), but a brittle material shows negligible elongation as shown in figure 15.1(b). A general definition of a ductile material is given by *"A material which can be drawn into wires is a ductile material and a material which cannot be drawn into wires is a brittle material."*

Example of ductile materials are gold (most ductile), silver, copper, aluminium, steel etc.

Common examples of brittle materials are chalk, marble, concrete, brick work, cast iron etc.

Majority of engineering components such as crank shaft, connecting rod, pins, shafts, couplings etc. are made from ductile materials. But then there are various components which require high strength in compression such as beds of machines, cylinder blocks of engines, brittle material as cast iron is used.

$$T = 400 \text{ Nm}$$

$$T_e = \text{Equivalent twisting moment}$$

$$= \sqrt{M^2 + T^2} = 806.225 \text{ Nm}$$

$$\text{FOS} = 1.5$$

$$\tau_{\text{allowable}} = \frac{275}{FOS} = \frac{\tau_y}{FOS}$$

$$= 183.33 \text{ MPa}$$

$$T_e = \frac{\pi}{16} d^3 T_a$$

$$806.275 \times 10^3 = \frac{\pi}{16} \times d^3 \times 183.33$$

$$d^3 = 22.396 \times 10^3$$

Shaft diameter, $d = 28.2$ mm

Objective Type Questions

Q.1 Graphical representation of which one of the following theories is an ellipse
(a) Maximum shear stress theory
(b) Maximum principal strain theory
(c) Distortion energy theory
(d) None of these

Q.2 Which one of the following theories gives conservative design of a component
(a) Maximum principal stress theory
(b) Maximum shear stress theory
(c) Shear strain energy theory
(d) None of these

Q.3 Principal stresses at a point are $+p$, $-p$, 0, using the shear strain energy FOS is $\sqrt{3}$, what is σ_{yp}
(a) $\sqrt{3}p$ (b) $3p$
(c) $1.5p$ (d) None of these

Q.4 A thick cylinder of outer radius two times the inner radius. Internal pressure is p. For this material σ_{yp} = 300 MPa, FOS = 2, using the maximum principal stress theory, what is the maximum value of p
(a) 150 MPa (b) 120 MPa
(c) 90 MPa (d) 75 MPa

Q.5 A shaft is subjected to a twisting moment such that maximum shear stress on surface of shaft is 100 MPa. Poisson's ratio of a material is 0.3. If σ_{yp} is 270, what is FOS as per maximum principal strain theory
(a) 3.857 (b) 2.7
(c) 2.077 (d) 1.732

Q.6 A shaft is subjected to a torque and an axial compressive force. Shear stress due to torque is 30 MPa and compressive stress due to force is 80 MPa. If σ_{yp} = 270 MPa, and ν = 0.3, what is FOS as per maximum shear stress theory
(a) 2.7 (b) 1.58
(c) 1.50 (d) None of these

Q.7 A thin cylindrical shell $\sigma_c = 30p$, $\sigma_a = 15p$, $\sigma_r = -p$. If σ_{yp} = 300 MPa, what is p as per maximum principal strain theory, if ν = 0.3
(a) 11.90 MPa (b) 11.63 MPa
(c) 11.89 MPa (d) None of these

Q.8 A distortion energy theory of failure if $\frac{p_1}{\sigma_{yp}} = x$, $\frac{p_2}{\sigma_{yp}} = y$, Poisson's ratio is 0.3, what is the equation of ellipse for graphical representation

Explanations

1. (c)
Distortion energy theory is graphically represented by an ellipse.

2. (b)
Maximum shear stress theory gives conservative design of a component.

3. (b)

$$(p^2 + p^2 + p^2) \le \left(\frac{\sigma_{yp}}{FOS}\right)^2$$

$$3p^2 = \left(\frac{\sigma_{yp}}{FOS}\right)^2$$

$$\sqrt{3}\,p = \frac{\sigma_{yp}}{FOS} = \frac{\sigma_{yp}}{\sqrt{3}}$$

$$\sigma_{yp} = 3p$$

4. (c)

$$\sigma_c = p\frac{R_2^2 + R_1^2}{R_2^2 - R_1^2}$$

$$= p \times \frac{5}{3} = \frac{300}{2} = 150$$

$$p = 90 \text{ MPa}$$

5. (c)

$$(p_1 - \nu p_2) = \frac{\sigma_{yp}}{FOS}$$

$$[100 - 0.3(-100)] = 130 = \frac{270}{FOS}$$

$$FOS = \frac{270}{130} = 2.077$$

6. (a)

$$\frac{\sigma}{2} = 40 \text{ MPa}, \tau = 30 \text{ MPa}$$

$$\tau_{max} = \sqrt{40^2 + 30^2} = 50 \text{ MPa}$$

$$FOS = \frac{\sigma_{yp}}{2} \times \frac{1}{\tau_{max}} = \frac{135}{50} = 2.7$$

7. (b)

$$p[30 - 0.3(15 - 1)] = 300$$

$$p[25.8] = 300$$

$$p = 11.628 \text{ N/mm}^2$$

8. (a)

$$\frac{p_1}{\sigma_{yp}} = x, \frac{p_2}{\sigma_{yp}} = y$$

Distortion energy theory, graphical representation

$$x^2 + y^2 - xy \le 1$$

9. (c)

$$\sigma_1 = 100 \text{ MPa}$$

$$\sigma_2 = 100 \text{ MPa}$$

$$\sigma_3 = 0$$

$$\tau_{max} = \frac{100}{2} = 50$$

$$\sigma_y = 200 \text{ MPa}$$

$$\frac{\sigma_y}{2} = 100 = \text{shear stress}$$

$$n_T = \frac{100}{50} = 2 \quad \text{...(i)}$$

Von Mises Theory

$$\sigma_a^2 - \sigma_1\sigma_2 + \sigma_2^2 = \left(\frac{\sigma_y}{n_v}\right)^2$$

$$100^2 = \left(\frac{200}{n_v}\right)^2$$

$$\text{or} \quad 100 = \frac{200}{n_v}$$

$$n_v = 2 \quad \text{...(ii)}$$

10. (d)
Principal stress

$$+1.5\,\sigma, +\sigma, -0.5\sigma$$

$$\sigma_e = 210 \text{ MPa}$$

$$\varepsilon_{max} = \frac{1.5\sigma}{E} - \frac{0.3\sigma}{E} + \frac{0.3 \times 0.5\sigma}{E}$$

General Questions

CHAPTER 17

Question 17.1 A thin cylinder is turning about its axis. Find the safe number of revolutions for a rotor of 3 meters in diameter if the hoop stress is not to exceed 13200 kg/cm^2. Take density as 6500 kg/cm^3.

[ME, ESE 2015 : 10 Marks]

Solution:

$$\sigma_{c\,max} = \frac{\rho\omega^2}{g}\left(R^2\right)\left(3k_1 - k_2\right), \quad \rho \text{ is density}$$

where,

$$k_1 = \frac{3-2\nu}{8(1-\nu)}, \quad \text{where } \nu \text{ is Poisson's ratio}$$

$$k_2 = \frac{1+2\nu}{8(1-\nu)}$$

$$3k_1 = \frac{9-6\nu}{8(1-\nu)}$$

$$3k_1 - k_2 = \frac{9-6\nu-1-2\nu}{8(1-\nu)} = \frac{8-8\nu}{8(1-\nu)} = 1$$

$$\sigma_{c\,max} = \frac{\rho\omega^2}{g} \times R^2 \quad \text{where R is radius of thin cylinder}$$

$$\rho = \frac{6500\,\text{kg}}{\text{m}^3}$$

$$R = 1.5 \text{ m}$$

$$g = 9.8 \text{ m/s}^2$$

$$\sigma_{c\,max} = 1300 \times 10^4 \text{ kg/m}^2$$

$$1300 \times 10^4 = 6500 \times \frac{\omega^2}{9.8} \times 1.5^2$$

$$\omega^2 = \frac{1300 \times 10^4 \times 9.8}{6500 \times 2.25} = 8711.11$$

$$\omega = 93.3 \text{ radian/second}$$

$$N_{safe} = \frac{93.3 \times 60}{2\pi} = 891.27 \text{ RPM}$$

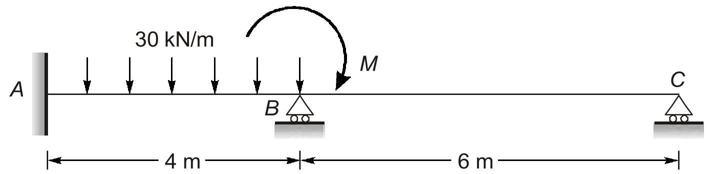


Fig. 17.7

The value of support reaction (in kN) at *B* should be equal to ________ .

[CE, GATE : 2017 (set-1)]

Solution: Say *M* is not applied initially.

$$8\,M_A + 4M_B = -\frac{30\times 4^3}{4},$$ where M_A, M_B are support moments, $M_C = 0$

$$8M_A + 4M_B = -480 \quad \text{...(i)}$$

$$4M_A + 20M_B = \frac{-30\times 4^3}{4} = -480 \quad \text{....(ii)}$$

$$8M_A + 40M_B = -960 \quad \text{...(iii)}$$

$$-36\,M_B = +480$$

$$M_B = -13.33 \text{ kNm}$$

$$M_A = -\frac{480}{4} - 5\,M_B$$

$$= -120 - 5\,(-13.33)$$

$$= -120 + 66.66 = -53.33 \text{ kNm}$$

$$M_C = 0$$

Putting, $M = +13.33$ kNm, no rotation at B

Moments about *A*

$$R_B \times 4 - 13.33 - 30 \times 4 \times 2 = -53.333$$

$$4R_B = +200$$

$$R_B = 50 \text{ kN}\uparrow$$

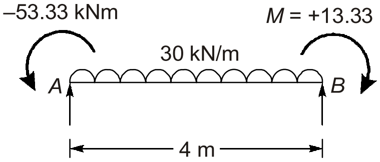


Fig. 17.8

Objective Type Questions

Q.1 Which of the following are examples of indeterminate structures?
1. Fixed beam
2. Continuous beam
3. Two-hinged arch
4. Beam overhanging on both sides

Select the correct answer using the codes given below:

(a) 1, 2 and 3 only (b) 1, 2 and 4 only
(c) 1, 3 and 4 only (d) 2, 3 and 4 only

[CE ESE (pre) : 2017]

Q.2 The first moment of area about the axis of bending for a beam cross-section is
(a) moment of inertia
(b) section modulus
(c) shape factor
(d) polar moment of inertia

[CE, GATE : 2017]

Q.3 Which of the bearing given below SHOULD NOT be subjected to a thrust load?